Shearfree Condition and dynamical Instability in $f(R, T)$ gravity

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Abstract

The implications of shearfree condition on instability range of anisotropic fluid in $f(R, T)$ are studied in this manuscript. A viable $f(R, T)$ model is chosen to arrive at stability criterion, where R is Ricci scalar and T is the trace of energy momentum tensor. The evolution of spherical star is explored by employing perturbation scheme on modified field equations and contracted Bianchi identities in $f(R, T)$. The effect of imposed shearfree condition on collapse equation and adiabatic index Γ is studied in Newtonian and post-Newtonian regimes.

Keywords: Shearfree condition; $f(R, T)$ gravity; Dynamical equations; Instability range; Adiabatic index.

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1 Introduction

In a recent work [1], we have studied the effect of anisotropic fluid on dynamical instability of spherically symmetric collapsing star in $f(R,T)$ theory. Herein, we plan to explore the instability range of anisotropic spherically symmetric stars, considering shearfree condition. The role of shear tensor in evolution of gravitating objects and consequences of shearfree condition has been studied extensively. Collins and Wainwright [2] studied the impact of shear on general relativistic cosmological and stellar models. Herrera et al. [3, 4] worked out the homology and shearfree conditions for dissipative and radiative gravitational evolution.

The features of gravitational evolution and its final outcome is of great importance in view of General Relativity (GR) as well as in modified theories of gravity. Shearfree collapse accounting heat flow is discussed in [5], it is established that shear plays critical role in gravitational evolution and may lead to the formation of naked singularities [6]. It is mentioned in [6] that occurrence of shearing effects near collapsing stars detains the apparent horizon leading to formation of naked singularity. However vanishing shear give rise to the formation of apparent horizon and so the evolving cloud ends in a black hole (BH). Thus, relevance of shear tensor in structure formation and its impressions on dynamical instability range of self gravitating body is well motivated direction of study.

Stars shine by consuming their nuclear fuel, continuous fuel consumption cause imbalance between inward acting gravitational pull and outward drawn pressure giving rise to collapse [7]. The outcome of gravitational evolution is size dependent as well as other physical aspects [8] such as isotopy, anisotropy, shear, radiation, dissipation etc. In comparison to the stars of mass around one solar masses, massive stars tend to lose nuclear fuel more rapidly and so more unstable. Pressure to density ratio naming adiabatic index, denoted by Γ is utile in estimation of stability/instability range of stars. Chandrasekhar [9] explored the instability range of spherical stars in terms of Γ.

Herrera and his contemporaries [10]-[16] contributed majorally in addressing the instability problem in general relativity (GR), accompanying various situations, i.e., isotopy, anisotropy, shearfree condition, radiation, dissipation, expansionfree condition, shearing expansionfree fluids. In order to achieve the more precise and generic description of universe, the dark energy components are incorporated by introducing modified theories of gravity. Modified theories are significant in advancements towards accelerated

expansion of the universe and to present corrections to GR on large scales. The modifications are introduced in Einstein Hilbert (EH) action by inducing minimal or non-minimal coupling of matter and geometry [17].

Dynamical analysis of self gravitating sources in modified theories of gravity has been discussed extensively in recent years. The null dust non-static exact solutions in $f(R)$ gravity are studied in [18], Cembranos et al. [19] studied the evolution of gravitating sources in the presence of dust fluid. Instability range of spherically and axially symmetric anisotropic stars has been established in context of $f(R)$ gravity [20, 21], concluding that deviations from spherical symmetry complicates the subsequent evolution.

Harko et al. [22] presented the $f(R,T)$ theory of gravity as another alternate to GR and generalization of $f(R)$ theory representing non-minimal matter to geometry coupling. The action in $f(R,T)$ gravity includes arbitrary function of Ricci scalar R and trace of energy momentum tensor T to take into account the exotic matter. After the introduction of $f(R,T)$ gravity, its cosmological and thermodynamic implications were widely studied [23]-[30] including the energy conditions. Recently, we have studied the evolution of anisotropic gravitating source with zero expansion [31]. Herein, we are interested in exploration of shearfree condition implications on spherically symmetric gravitating source in $f(R,T)$ gravity.

The modified action in $f(R,T)$ is as follows [22]

$$\int dx^4 \sqrt{-g}[\frac{f(R,T)}{16\pi G} + \mathcal{L}_{(m)}],\tag{1.1}$$

where $\mathcal{L}_{(m)}$ denote matter Lagrangian, and g represents the metric tensor. Various choices of $\mathcal{L}_{(m)}$ can be taken into account, each of which leads to a specific form of fluid. Many people worked out this problem in GR and modified theories of gravity, stability of general relativistic dissipative axially symmetric and spherically symmetric with shearfree condition has been established in [32, 33]. Dynamical analysis of shearfree spherically symmetric sources in $f(R)$ gravity is presented in [34].

The manuscript arrangement is: Section **2** comprises of modified dynamical equations in $f(R,T)$ gravity. Section **3** includes model under consideration, perturbation scheme and corresponding collapse equation alongwith shear-free condition in Newtonian and post Newtonian eras. Section **4** contains concluding remarks followed by an appendix.

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2 Dynamical Equations in $f(R,T)$

In order to study the implications of shearfree condition on evolution of spherically symmetric anisotropic sources, modified field equations in $f(R,T)$ gravity are formulated by varying action (1.1) with the metric g_{uv}. Here, we have taken $\mathcal{L}_{(m)} = \rho$ [27], for this choice of $\mathcal{L}_{(m)}$ modified field equations in $f(R,T)$ gravity takes following form

$$
\begin{aligned}
G_{uv} &= \frac{1}{f_R} \left[(f_T + 1)T_{uv}^{(m)} - \rho g_{uv} f_T + \frac{f - Rf_R}{2} g_{uv} \right. \\
&+ \left. (\nabla_u \nabla_v - g_{uv}\Box)f_R \right].
\end{aligned}
\tag{2.2}
$$

Here $T_{uv}^{(m)}$ is energy momentum tensor for usual matter taken to be locally anisotropic.

The three dimensional spherical boundary surface Σ is considered that constitutes two regions named as interior and exterior spacetimes. The line element for region inside the boundary Σ is

$$
ds_-^2 = A^2(t,r)dt^2 - B^2(t,r)dr^2 - C^2(t,r)(d\theta^2 + \sin^2\theta d\phi^2).
\tag{2.3}
$$

The line element for region beyond Σ is [1]

$$
ds_+^2 = \left(1 - \frac{2M}{r} \right)d\nu^2 + 2drd\nu - r^2(d\theta^2 + \sin^2\theta d\phi^2),
\tag{2.4}
$$

where ν is retarded time and M denote the total mass.

The expression for anisotropic energy momentum tensor $T_{uv}^{(m)}$ is given by

$$
T_{uv}^{(m)} = (\rho + p_\perp)V_u V_v - p_\perp g_{uv} + (p_r - p_\perp)\chi_u \chi_v,
\tag{2.5}
$$

where ρ is energy density, V_u describes four-velocity of the fluid, χ_u is radial four vector, p_r and $p_\perp$ represent the radial and tangential pressure respectively. These physical quantities are linked as

$$
V^u = A^{-1}\delta_0^u, \quad V^u V_u = 1, \quad \chi^u = B^{-1}\delta_1^u, \quad \chi^u \chi_u = -1.
\tag{2.6}
$$

The shear tensor denoted by σ_{uv} is defined as

$$
\sigma_{uv} = V_{(u;v)} - a_{(u}V_{v)} - \frac{1}{3}\Theta(g_{uv} - V_u V_v),
\tag{2.7}
$$

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where a_u is four acceleration and Θ is expansion scalar, given by

$$a_u = V_{(u;v)}V^v, \quad \Theta = V^u_{;u}. \tag{2.8}$$

Components of shear tensor are found by variation of Eq. (2.7) and these are used to find expression for shear scalar in the following form

$$\sigma = \frac{1}{A}\left(\frac{\dot{B}}{B} - \frac{\dot{C}}{C}\right), \tag{2.9}$$

where dot and prime indicate time and radial derivatives respectively. From shearfree condition we arrive at vanishing shear scalar, i.e., $\sigma = 0$, implying $\frac{\dot{B}}{B} = \frac{\dot{C}}{C}$.

It is worth mentioning here that the expansion scalar and a scalar function described in terms of Weyl tensor and the anisotropy of pressure controls the departure from shearfree condition. Such function is related to the Tolman mass and appears in a natural way in the orthogonal splitting of the Reimann tensor [35]. It is obvious that pressure anisotropy and density inhomogeneities have extensive implications on stability of the shearfree condition but it is not intuitively clear that their specific combination affects stability [33]. Generically shearfree condition remains unstable against the presence of pressure anisotropy. Alternatively, one can consider such a case that pressure anisotropy and density inhomogeneity are present in a way that the scalar function appearing in orthogonal splitting of Reimann tensor vanishes, implying non-homogeneous anisotropic stable shear free flow. Since we are dealing with fluid evolving under shearfree condition, so we shall make use of this condition while evaluating the components of field equations and also in conservation equations.

The components of modified Einstein tensor are

$$G_{00} = \frac{1}{f_R}\left[\rho + \frac{f - Rf_R}{2} + \frac{f_R''}{B^2} - \frac{3\dot{f}_R}{A^2}\frac{\dot{B}}{B} - \frac{f_R'}{B^2}\left(\frac{B'}{B} - \frac{2C'}{C}\right)\right], \quad (2.10)$$

$$G_{01} = \frac{1}{f_R}\left[\dot{f}_R{}' - \frac{A'}{A}\dot{f}_R - \frac{\dot{B}}{B}f_R'\right], \quad (2.11)$$

$$G_{11} = \frac{1}{f_R}\left[p_r + (\rho + p_r)f_T - \frac{f - Rf_R}{2} + \frac{\ddot{f}_R}{A^2} - \frac{\dot{f}_R}{A^2}\left(\frac{\dot{A}}{A} - \frac{2\dot{C}}{C}\right)\right.$$
$$\left. - \frac{f_R'}{B^2}\left(\frac{A'}{A} + \frac{2C'}{C}\right)\right], \quad (2.12)$$

$$G_{22} = \frac{1}{f_R}\left[p_\perp + (\rho + p_\perp)f_T - \frac{f - Rf_R}{2} + \frac{\ddot{f}_R}{A^2} - \frac{f_R''}{B^2} - \frac{\dot{f}_R}{A^2}\left(\frac{\dot{A}}{A}\right.\right.$$
$$\left.\left. - \frac{2\dot{B}}{B}\right) - \frac{f_R'}{B^2}\left(\frac{A'}{A} - \frac{B'}{B} + \frac{C'}{C}\right)\right]. \quad (2.13)$$

The dynamical equations extracted from the conservation laws are vital in the study of stellar evolution. The conservation of full field equations is considered to incorporate the non-vanishing divergence terms, Bianchi identities are

$$G^{uv}_{;v}V_u = 0, \quad G^{uv}_{;v}\chi_u = 0, \quad (2.14)$$

on simplification of Eq.(2.14), we have dynamical equations as follows

$$\dot{\rho} - \rho\frac{\dot{f}_R}{f_R} + [1 + f_T](3\rho + p_r + 2p_\perp)\frac{\dot{B}}{B} + Z_1(r,t) = 0, \quad (2.15)$$

$$(\rho + p_r)f_T' + (1 + f_T)\left\{p_r' + \rho\frac{A'}{A} + p_r\left(\frac{A'}{A} + 2\frac{C'}{C} - \frac{f_R'}{f_R}\right) - 2p_\perp\frac{C'}{C}\right\}$$
$$+ f_T\left(\rho' - \frac{f_R'}{f_R}\right) + Z_2(r,t) = 0, \quad (2.16)$$

where $Z_1(r,t)$ and $Z_2(r,t)$ are provided in appendix as Eqs.(5.1) and (5.2) respectively. Deviations from equilibrium in conservation equations with the time transition leads to the stellar evolution, perturbation approach is devised to the estimate the instability range.

3 Perturbation Scheme and Shearfree Condition

We consider a particular $f(R,T)$ model of the form

$$f(R,T) = R + \alpha R^2 + \lambda T, \tag{3.17}$$

where α and λ can be any positive constants. Perturbation approach is utilized to estimate the instability range of spherical star with shear free condition. This scheme is utile in determination of more generic analytical constraints on collapse equation, rather to establish dynamical analysis of special cases numerically. Also, field equations are highly nonlinear differential equations, in such scenario application of perturbation lead to beneficial observations.

It is assumed that initially all quantities are independent of time and with the passage of time perturbed form depends on both time and radial coordinates. Taking $0 < \varepsilon \ll 1$, the physical quantities and their perturbed form can be arranged as

$$
\begin{aligned}
A(t,r) &= A_0(r) + \varepsilon D(t)a(r), & (3.18)\\
B(t,r) &= B_0(r) + \varepsilon D(t)b(r), & (3.19)\\
C(t,r) &= C_0(r) + \varepsilon D(t)\bar{c}(r), & (3.20)\\
\rho(t,r) &= \rho_0(r) + \varepsilon \bar{\rho}(t,r), & (3.21)\\
p_r(t,r) &= p_{r0}(r) + \varepsilon \bar{p}_r(t,r), & (3.22)\\
p_\perp(t,r) &= p_{\perp 0}(r) + \varepsilon \bar{p}_\perp(t,r), & (3.23)\\
m(t,r) &= m_0(r) + \varepsilon \bar{m}(t,r), & (3.24)\\
R(t,r) &= R_0(r) + \varepsilon D_1(t)e_1(r), & (3.25)\\
T(t,r) &= T_0(r) + \varepsilon D_2(t)e_2(r), & (3.26)\\
f(R,T) &= [R_0(r) + \alpha R_0^2(r) + \lambda T_0] + \varepsilon(D_1(t)e_1(r)[1 \\
&\quad + 2\alpha R_0(r)] + D_2(t)e_2(r)), & (3.27)\\
f_R &= 1 + 2\alpha R_0(r) + \varepsilon 2\alpha D_1(t)e_1(r), & (3.28)\\
f_T &= \lambda. & (3.29)
\end{aligned}
$$

Considering Schwarzschild coordinate $C_0 = r$ and implementing perturbation scheme on vanishing shear scalar implies

$$\frac{b}{B_0} = \frac{\bar{c}}{r}. \tag{3.30}$$

Using Eqs. (3.18)-(3.29) and (3.30) in dynamical equations i.e., Eqs.(2.15) and (2.16) leads to following expressions

$$\dot{\bar{\rho}} + \left[\frac{2e\rho_0}{Y} + \lambda_1\frac{\bar{c}}{r}(2\rho_0 + p_{r0} + 4p_{\perp 0}) + YZ_{1p}\right]\dot{D} = 0, \tag{3.31}$$

$$\lambda_1\left\{\bar{p_r}' + \bar{\rho}\frac{A_0'}{A_0} + \bar{p_r}\left(\frac{A_0'}{A_0} + \frac{2}{r} - \frac{2\alpha R_0'}{Y}\right) - \frac{2\bar{p_\perp}}{r}\right\} + \lambda\bar{\rho}' + 2\alpha\ddot{D}\left[\frac{1}{A_0^2}\left(e'\right.\right.$$
$$\left.+2e\frac{B_0'}{B_0} - \frac{\bar{c}}{r}R_0'\right) + B_0^2(Y)\left\{\frac{e}{B_0^2Y}\right\}'\right] + D\left[\lambda_1[(\frac{a}{A_0})'(\rho_0 + p_{r0})\right.$$
$$\left.-2(p_{r0} + p_{\perp 0})(\frac{\bar{c}}{r})'] - \frac{2\alpha}{Y}\left\{\lambda_1\left(p_{r0}' + \rho_0\frac{A_0'}{A_0} + p_{r0}\left(\frac{A_0'}{A_0} - \frac{2\alpha R_0'}{Y} + \frac{2}{r}\right)\right)\right\}\right.$$
$$\left.+\lambda\left(e' + e[\rho_0' - \frac{2\alpha R_0'}{Y}]\right) + YZ_{2p}\right] = 0, \tag{3.32}$$

where Z_{1p} and Z_{2p} are given in appendix, for the sake of simplicity we put Y in place of $1 + 2\alpha R_0$ and $\lambda_1 = \lambda + 1$, assuming that $D_1 = D_2 = D$ and $e_1 = e_2 = e$. Above mentioned perturbed dynamical equations and perturbed field equations shall be used to arrive at perturbed physical quantities such as $\bar{\rho}, \bar{p_r}$ and $\bar{p_\perp}$.

The expression for $\bar{\rho}$ can be found from Eq.(3.31), as follows.

$$\bar{\rho} = -\left[\frac{2e\rho_0}{Y} + \frac{\bar{c}}{r}(3\rho_0 + p_{r0} + 4p_{\perp 0}) + YZ_{1p}\right]D. \tag{3.33}$$

The Harrison-Wheeler type equation of state relate $\bar{\rho}$ and $\bar{p_r}$, given by

$$\bar{p_r} = \Gamma\frac{p_{r0}}{\rho_0 + p_{r0}}\bar{\rho}. \tag{3.34}$$

Putting $\bar{\rho}$ from Eq.(3.33) in Eq.(3.34), we found

$$\bar{p_r} = -\Gamma\frac{p_{r0}}{\rho_0 + p_{r0}}\left[\frac{2e\rho_0}{Y} + \lambda_1\frac{\bar{c}}{r}(3\rho_0 + p_{r0} + 4p_{\perp 0}) + YZ_{1p}\right]D. \tag{3.35}$$

Perturbed form of field equation (2.13) yields expression for $\bar{p_\perp}$, that turns out to be

$$\bar{p_\perp} = \{\frac{Y\bar{c}}{r} - 2\alpha e\}\frac{\ddot{D}}{A_0^2} - \frac{\lambda\bar{\rho}}{\lambda_1} + \left\{\left(p_{\perp 0} - \frac{\lambda}{\lambda_1}\rho_0\right)\frac{2\alpha e}{Y} + \frac{Z_3}{\lambda_1}\right\}D, \tag{3.36}$$

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Z_3 is effective part of the field equation given in appendix as Eq.(5.5).

Substitution of $\bar{\rho}, \bar{p}_r$ and $\bar{p}_\perp$ from Eqs.(3.33), (3.35) and (3.36) in Eq.(3.32) leads to collapse equation as under

$$\ddot{D}\left[\frac{2\alpha}{A_0^2 Y}\left\{e' + 2e\frac{B_0'}{B_0} - \frac{\bar{c}}{r}R_0'\right\} - 2\alpha B_0^2\left\{\frac{e}{B_0^2 Y}\right\}' + \frac{1}{A_0^2}\{\frac{Y\bar{c}}{r} - 2\alpha e\}\right]$$

$$+D\left[\frac{1}{Y}\left\{\lambda_1\left((\rho_0 + p_{r0})\left(\frac{a}{A_0}\right)' - 2(\rho_0 + p_{\perp 0})\left(\frac{\bar{c}}{r}\right)'\right) - \frac{2\alpha}{Y}\{\lambda(e' - \rho_0')\right.$$

$$\left. -\frac{2\alpha R_0'}{Y}\right) + \lambda_1\left(e'p_{r0} + e[p_{r0}' + \rho_0\frac{A_0'}{A_0} + p_{r0}(\frac{A_0'}{A_0} + \frac{2}{r} - \frac{2\alpha R_0'}{Y})]\right)\right\} - (\lambda$$

$$+\lambda_1\Gamma\frac{p_{r0}}{\rho_0 + p_{r0}}\right)\left\{\rho_0\frac{2e}{Y} + \lambda_1\frac{\bar{c}}{r}(3\rho_0 + p_{r0} + 4p_{\perp 0}) + YZ_{1p}\right\}_{,1} + \left\{\frac{A_0'}{A_0}\right.$$

$$+\frac{2}{r}\frac{\lambda}{\lambda_1} + \Gamma\frac{p_{r0}}{\rho_0 + p_{r0}}(\frac{A_0'}{A_0} + \frac{2}{r} - \frac{2\alpha R_0'}{Y}) + \lambda_1\left(\Gamma\frac{p_{r0}}{\rho_0 + p_{r0}}\right)'\right\}\left\{\frac{2e\rho_0}{Y}\right.$$

$$\left. +\lambda_1\frac{\bar{c}}{r}(3\rho_0 + p_{r0} + 4p_{\perp 0}) + (Y)Z_{1p}\right\} + \frac{2}{r}\frac{1}{\lambda_1}Z_3\right\} + Z_{2p}\right] = 0. \qquad (3.37)$$

Matching conditions at boundary surface together with perturbed form of Eq.(2.13) can be written in the simplified form as follows

$$\ddot{D}(t) - Z_4(r)D(t) = 0, \qquad (3.38)$$

provided that

$$Z_4 = \frac{rA_0^2}{Y\bar{c} - 2\alpha er}\left[\frac{2\alpha e}{Y}p_{\perp 0} + \lambda\frac{\bar{c}}{r}(3\rho_0 + p_{r0} + 4p_{\perp 0}) + YZ_{1p} + \frac{Z_3}{\lambda_1}\right]. \qquad (3.39)$$

The valid solution of Eq.(3.38) turns out to be

$$D(t) = -e^{\sqrt{Z_4}t}. \qquad (3.40)$$

The terms of Z_4 must be constrained in a way that all terms maintain positivity. Impact of shear-free condition on dynamical instability of N and pN regimes is covered in following subsections.

3.1 Newtonian Regime

In order to establish instability range in Newtonian era, we set $\rho_0 \gg p_{r0}$, $\rho_0 \gg p_{\perp 0}$ and $A_0 = 1$, $B_0 = 1$. Insertion of these assumptions and Eq.(3.40)

in Eq.(3.37) leads to the instability condition, relating usual matter and dark source contribution as under

$$\Gamma < \frac{Z_4 X_3 + X_4 + \lambda \rho_0 (X_2 + YZ_{1p(N)})_{,1} + X_1 X_2 - \frac{2}{r\lambda_1} Z_{3(N)} + YZ_{2p(N)}}{\lambda_1 p_{r0} X_2' + \left\{ p_{r0} \left(\frac{2\alpha R_0'}{Y} - \frac{2}{r} \right) \right\} X_2},$$

(3.41)

where

$$X_1 = (\lambda \rho_0' + \frac{2\lambda}{r\lambda_1}), \quad X_2 = \frac{2e}{Y} + 3\lambda_1 b, \quad X_3 = -2\alpha^2 b R_0' + Yb,$$

$$Z_4 = \lambda_1 \left[\rho_0 a' + 2(p_{r0} + p_{\perp 0})b' \right] + \frac{2\alpha}{Y} \left[\lambda \left(\frac{2\alpha R_0'}{Y} - \rho_0' + e' \right) \right.$$

$$\left. + \lambda_1 \left\{ p_{r0} + e[p_{r0}' + p_{r0} \left(\frac{2}{r} - \frac{2\alpha R_0'}{Y} \right)] \right\} \right].$$

The quantities $Z_{1p(N)}$ and $Z_{2p(N)}$ are terms of Z_{1p} and Z_{2p} belonging to Newtonian era. The gravitating source remains stable in Newtonian approximation until the inequality for Γ satisfies, for which following constraints must be accomplished.

$$2\alpha R_0' < Y, \quad \frac{2\alpha R_0'}{Y} > \rho_0' - e'$$

The case when $\alpha \to 0$ and $\lambda \to 0$ leads to GR corrections and results for $f(R)$ can be retrieved by setting $\lambda \to 0$.

3.2 Post Newtonian Regime

We assume $A_0 = 1 - \frac{m_0}{r}$ and $B_0 = 1 + \frac{m_0}{r}$ to evaluate stability condition in pN regime. On substitution of these assumptions in Eq. (3.37), we have following inequality for Γ to be fulfilled for stability range.

$$\Gamma < \frac{Z_4 X_5 + X_6 + \lambda \rho_0 (X_7 + YZ_{1p(PN)})_{,1} + X_8 X_7 - \frac{2}{r\lambda_1} Z_{3(PN)} + YZ_{2p(PN)}}{\lambda_1 p_{r0} X_7' + \left\{ p_{r0} \left(\frac{m_0}{r(r-m_0)} + \frac{2\alpha R_0'}{Y} + \frac{2}{r} \right) \right\} X_7},$$

(3.42)

where

$$
\begin{aligned}
X_5 &= \frac{2\alpha r^2}{(r-m_0)^2}\left\{e' - \frac{r}{r+m_0}\left(bR_0' + 2e\frac{m_0}{r}\right)\right\} + Y\left[\frac{r^2}{(r-m_0)^2}\left\{2\alpha e\right.\right. \\
&\quad \left.\left. -Y\frac{\bar{c}}{r}\right\} - \frac{2\alpha(r+m_0)^2}{r^2}\left\{\frac{er^2}{Y(r+m_0)^2}\right\}'\right], \\
X_6 &= \lambda_1\left\{\rho_0\left(\frac{ar}{r-m_0}\right)' - 2(p_{r0} + p_{\perp0})b'\right\} - \frac{2\alpha}{Y}\left[(\lambda_1 p_{r0} + \lambda)e'\right. \\
&\quad \left. +e\left\{\lambda_1(p_{r0}' + \frac{\rho_0 m_0}{r(r-m_0)}) + p_{r0}\left(\frac{2}{r} - \frac{2\alpha R_0'}{Y}\right)\right\} - \lambda\left(\rho_0' - \frac{2\alpha R_0'}{Y}\right)\right], \\
X_7 &= \frac{2e}{Y} + \lambda_1 b\left(2 + \frac{r}{r+m_0}\right), \quad X_8 = \left(\frac{m_0}{r(r-m_0)} + \frac{2\alpha R_0'}{Y} + \frac{2\lambda}{\lambda_1 r} + \lambda\rho_0'\right).
\end{aligned}
$$

$Z_{1p(PN)}$ and $Z_{2p(PN)}$ are terms of Z_{1p} and Z_{2p} that lie in post-Newtonian era. The above inequality (3.42) holds for positive definite terms and describe the stability range of subsequent evolution. The positivity of each term appearing in (3.42) leads to following restrictions

$$
\frac{r}{r+m_0}(bR_0' + \frac{2em_0}{r}) < e', \quad 2\alpha e - Yb > \frac{(r^2 - m_0^2)^2}{r^4}\left\{\frac{er^2}{Y(r+m_0)^2}\right\}'
$$

$$
\left(\frac{ar}{r-m_0}\right)' > 2(p_{r0} + p_{\perp0})b', \quad \rho_0' < \frac{2\alpha R_0'}{Y}.
$$

4   Concluding Remarks

In this manuscript, we carried out a study on implications of shear-free condition on stability of spherically symmetric anisotropic stars in $f(R,T)$. Our exploration regarding viability of the $f(R,T)$ model reveals that the selection of $f(R,T)$ model for dynamical analysis is constrained to the form $f(R,T) = f(R) + \lambda T$, where λ is arbitrary positive constant. The restriction on $f(R,T)$ form originates from the complexities of non-linear terms of trace in analytical formulation of field equations. The model under consideration is of the form $f(R,T) = R + \alpha R^2 + \lambda T$, representing a viable substitute to dark source and the exotic matter satisfying both viability criterion (positivity of radial derivatives upto second order).

In $f(R,T)$ gravity, the non-minimal matter geometry coupling include the terms of trace T in action (1.1) that is beneficent in the description

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of quantum effects or so-called exotic matter. The components of modified field equations together with the implementation of shear-free condition are developed in section **2**. Further conservation laws are considered to arrive at dynamical equations by means of Bianchi identities. These equations are utilized to estimate the variations in gravitating system with the passage of time.

The complexities of more generic analytical field equations are dealt by using linear perturbation of physical quantities. Perturbation scheme induce significant ease in the description of dynamical system, rather to present stability analysis by means of numerical simulations. The analytic approach we have employed here is more general and substantially important in explorations regarding structure formation. The perturbed shear-free condition together with the dynamical and field equations lead to the evolution equation, relating Γ with the usual and dark source terms. It is found that the induction of trace of energy momentum tenor in action (1.1) contributes positive addition to Γ, that slows down subsequent evolution considerably.

The outcome of gravitational evolution is size dependent as well as other physical aspects such as isotopy, anisotropy, shear, radiation, dissipation etc. The instability range for N and pN approximations is considered that impose some restrictions on the physical variables. It is observed that the terms appearing in Γ are less constrained for both the regimes (N and pN) in comparison to the anisotropic sources [1]. Thus, shear-free condition benefits in more stable anisotropic configurations. Corrections to GR and $f(R)$ establishments can be made by setting $\alpha \to 0$, $\lambda \to 0$ and $\lambda \to 0$ respectively. The local isotropy of the model can be settled by assuming $p_r = p_\perp = p$. The extension of this work for shearing expansion free evolution of anisotropic spherical and cylindrical sources is in process.

5 Appendix

$$
\begin{aligned}
Z_1(r,t) \;=\; & f_R A^2 \left[\left\{ \frac{1}{f_R A^2} \left(\frac{f - R f_R}{2} - \frac{3 \dot{f}_R}{A^2} \frac{\dot{B}}{B} - \frac{f'_R}{B^2} \left(\frac{B'}{B} - \frac{2C'}{C} \right) + \frac{f''_R}{B^2} \right) \right\}_{,0} \right. \\
& \left. + \left\{ \frac{1}{f_R A^2 B^2} \left(\dot{f}'_R - \frac{A'}{A} \dot{f}_R - \frac{\dot{B}}{B} f'_R \right) \right\}_{,1} \right] - \frac{\dot{f}_R}{A^2} \left\{ \left(3 \frac{\dot{B}}{B} \right)^2 + \frac{9 \dot{A}}{A} \frac{\dot{B}}{B} \right\} \\
& + \frac{3 \ddot{f}_R}{A^2} \frac{\dot{B}}{B} + \frac{\dot{A}}{A} (f - R f_R) - \frac{2 f'_R}{B^2} \left\{ \frac{\dot{A}}{A} \left(\frac{B'}{B} - \frac{C'}{C} \right) + \frac{\dot{B}}{B} \left(\frac{3A'}{A} - \frac{2C'}{C} \right. \right. \\
& \left. \left. + \frac{B'}{B} \right) \right\} + \frac{f''_R}{B^2} \left(\frac{2 \dot{A}}{A} + \frac{\dot{B}}{B} \right) + \frac{1}{B^2} \left(\dot{f}'_R - \frac{A'}{A} \dot{f}_R \right) \left(\frac{3A'}{A} + \frac{B'}{B} + \frac{2C'}{C} \right),
\end{aligned}
$$

$$(5.1)$$

$$
\begin{aligned}
Z_2(r,t) \;=\; & f_R B^2 \left[\left\{ \frac{1}{f_R A^2 B^2} \left(\dot{f}_R{}' - \frac{A'}{A} \dot{f}_R - \frac{\dot{B}}{B} f'_R \right) \right\}_{,0} + \left\{ \frac{1}{f_R B^2} \left(\frac{R f_R - f}{2} \right. \right. \right. \\
& \left. \left. \left. - \frac{\dot{f}_R}{A^2} \left(\frac{\dot{A}}{A} - \frac{2 \dot{C}}{C} \right) - \frac{f'_R}{B^2} \left(\frac{A'}{A} + \frac{2C'}{C} \right) + \frac{\ddot{f}_R}{A^2} \right) \right\}_{,1} \right] + (R f_R - f) \frac{B'}{B} \\
& - \frac{1}{A^2} \frac{\dot{f}_R}{A^2} \left\{ \frac{A'}{A} \left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} \right) + \frac{B'}{B} \left(\frac{\dot{A}}{A} - \frac{2 \dot{C}}{C} \right) \right\} + \left(\frac{\dot{A}}{A} + \frac{5 \dot{B}}{B} \right) \left(\dot{f}_R{}' \right. \\
& \left. - \frac{A'}{A} \dot{f}_R - \frac{\dot{B}}{B} f'_R \right) - \frac{f'_R}{B^2} \left\{ \frac{A'}{A} \left(\frac{A'}{A} + \frac{3B'}{B} \right) + \frac{2C'}{C} \left(\frac{3B'}{B} + \frac{C'}{C} \right) \right\} \\
& + \frac{\ddot{f}_R}{A^2} \left(\frac{A'}{A} + \frac{2B'}{B} \right) + \frac{f''_R}{B^2} \left(\frac{A'}{A} + \frac{2C'}{C} \right).
\end{aligned}
$$

$$(5.2)$$

$$Z_{1p} = 2\alpha A_0^2 \left[\frac{1}{A_0^2 B_0^2 Y} \left\{ e' - e\frac{A_0'}{A_0} - \frac{b}{B_0}R_0' \right\} \right]_{,1} + \frac{1}{Y}\left[e - [\lambda T_0 \right.$$

$$-\alpha R_0^2]\left(\frac{a}{A_0} + \frac{e}{Y} \right) + \frac{2\alpha}{B_0^2}\left\{ \left(\frac{B_0'}{B_0} - \frac{2}{r} \right)\left(e' - 2R_0'\left(\frac{a}{A_0} + \frac{b}{B_0} \right)\right.\right.$$

$$\left. + \frac{2\alpha e}{Y}R_0' \right) + R_0''\left(\frac{2a}{A_0} + \frac{b}{B_0} \right) - 2R_0'\left(\frac{b}{B_0}\left(\frac{2A_0'}{A_0} + \frac{B_0'}{B_0} + \frac{1}{r} \right)\right.$$

$$\left.\left.\left. + \frac{\bar{c}}{r}\left(\frac{A_0'}{A_0} - \frac{3}{r} \right)\right) + \left(e' - e\frac{A_0'}{A_0} \right)\left(\frac{3A_0'}{A_0} + \frac{B_0'}{B_0} + \frac{2}{r} \right)\right\} \right] \qquad (5.3)$$

$$Z_{2p} = B_0^2 Y\left[\frac{1}{B_0^2 Y}\left\{ e - \frac{2\alpha}{B_0^2}\left\{ \left(\frac{A_0'}{A_0} + \frac{2}{r} \right)\left(e - [\frac{2\alpha e}{Y} + \frac{4b}{B_0}]R_0' \right) + R_0'[\left(\frac{a}{A_0} \right)' \right.\right.\right.$$

$$\left.\left. + \left(\frac{\bar{c}}{r} \right)'] \right\} - [\lambda T_0 - \alpha R_0^2]\left(\frac{b}{B_0} + \frac{e}{Y} \right)\right\} \right]_{,1} + bB_0 Y\left[\frac{1}{B_0^2 Y}\left\{ \lambda T_0 - \alpha R_0^2 \right.$$

$$\left.\left. - \frac{4\alpha}{B_0^2}\left(\frac{A_0'}{A_0} + \frac{2}{r} \right)R_0' \right\} \right]_{,1} + \frac{2\alpha}{B_0^2}\left[R_0''\left\{ \left(\frac{a}{A_0} \right)' - 2\left(\frac{A_0'}{A_0} + \frac{2}{r} \right)\left(\frac{b}{B_0} + \frac{e}{Y} \right)\right.\right.$$

$$\left. + \left(\frac{\bar{c}}{r} \right)' \right\} - R_0'\left\{ \frac{A_0'}{A_0}\left[2\left(\frac{a}{A_0} \right)' + 3\left(\frac{b}{B_0} \right)' \right] + 3\frac{B_0'}{B_0}\left[\left(\frac{a}{A_0} \right)' + 2\left(\frac{\bar{c}}{r} \right)' \right] \right.$$

$$\left. + \frac{2}{r}\left[\left(3\frac{b}{B_0} \right)' + 2\left(\frac{\bar{c}}{r} \right)' \right] \right\} + \left(\frac{2b}{B_0}R_0' - e \right)\left\{ 3\frac{B_0'}{B_0}\left(\frac{A_0'}{A_0} + \frac{2}{r} \right) + \left(\frac{A_0'}{A_0} \right)^2 \right.$$

$$\left.\left. + \frac{2}{r^2} \right\} \right] + e\frac{B_0'}{B_0} - [\lambda T_0 - \alpha R_0^2]\left(\frac{b}{B_0} + \frac{2e}{Y}\frac{B_0'}{B_0} \right) \qquad (5.4)$$

$$Z_3 = \frac{Y}{B_0^2}\left[\frac{a''}{A_0} + \frac{\bar{c}''}{r} - \frac{A_0''}{A_0}\left(\frac{a}{A_0} + \frac{2b}{B_0} \right) + \frac{A_0'}{A_0}\left\{ \frac{2b}{B_0}\left(\frac{B_0'}{B_0} - \frac{1}{r} \right) + \left(\frac{\bar{c}}{r} \right)' \right.\right.$$

$$\left. - \left(\frac{b}{B_0} \right)' \right\} + \frac{B_0'}{B_0}\left\{ \frac{2bB_0'}{rB_0} - \left(\frac{a}{A_0} \right)' - \left(\frac{\bar{c}}{r} \right)' \right\} + \frac{1}{r}\left\{ \left(\frac{a}{A_0} \right)' - \left(\frac{b}{B_0} \right)' \right\} \right]$$

$$- \frac{2\alpha e}{Y}\left\{ \frac{\lambda T_0 - \alpha R_0^2}{2} - \frac{2\alpha}{B_0^2}\left(R_0'\left(\frac{A_0'}{A_0} - \frac{B_0'}{B_0} + \frac{1}{r} \right) - R_0'' \right) \right\} - \frac{2\alpha}{B_0^2}\left\{ e'' \right.$$

$$\left. + \frac{2b}{B_0}R_0'' + \left(\frac{A_0'}{A_0} - \frac{B_0'}{B_0} + \frac{1}{r} \right)\left(\frac{2b}{B_0}R_0' - e' \right) \right\} \qquad (5.5)$$

Acknowledgment

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